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Quantum mechanical description and noise analysis of the cyclotron resonance spectrometer–detector

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Abstract. The interaction between a radiation field and a system of electrons in cyclotron motion in a homogeneous magnetic field is described from a quantum mechanical viewpoint. With high magnetic fields the interaction can give a means of far infrared detection and high resolution spectral analysis. The sources of fluctuations that limit the ultimate detectivity are discussed. It is concluded that this limit is set by background room temperature radiation fluctuations at a level of approximately $1{\cdot}4 \times 10^{-16}$ W Hz$^{-1/2}$ when the interaction time is 10^{-7} s. With longer interaction times the limit of sensitivity may be well below 10^{-16} W Hz$^{-1/2}$.

1. Introduction

In a recent paper (Robinson 1970a) one of the authors proposed a means of far infrared radiation detection and spectral resolution based on free electron cyclotron resonance in a high magnetic field. A system of orbiting monoenergetic electrons brought into interaction with an oscillating electric field in an overmoded resonator has its energy redistributed, and observation of this redistribution gives a measure of the perturbing radiation field. Configurations of magnetic fields required for the production of the monoenergetic cyclotron oscillators have been discussed in detail (Robinson 1970a, Robinson and Szekeres 1970 and Robinson 1970b). Energy analysis after the interaction is achieved by unwinding the cyclotron motion with an inverse magnetic mirror field and selecting by means of bias electrodes those electrons that have acquired maximum energy from the radiation.

The previous calculation has shown that the energy acquired by an electron in cyclotron motion in a rotating electric field $E\exp(\mathrm{i}\omega t)$ can be written

$$W = W_1 + W_2 = \frac{q^2E^2\tau^2}{2m}\frac{\sin^2\Gamma}{\Gamma^2} + qEu_0\tau\sin(\Gamma+\phi)\frac{\sin\Gamma}{\Gamma} \tag{1}$$

where

$$\Gamma = (\omega-\omega_c)\frac{\tau}{2} \qquad \omega_c = \frac{qB}{m}.$$

Here q and m are the electronic charge and mass, respectively, B is the magnetic induction field, u_0 is the velocity of the orbiting particle before the interaction, τ is the duration of the interaction, ω and ω_c are the angular frequencies of the field and particle, respectively, and ϕ specifies the phase angle of the particle with respect to the field. It has been shown that while the phase averaged term $W_1 = (q^2E^2\tau^2/2m)(\sin^2\Gamma/\Gamma^2)$ can give a sensitive measure of E, the term $W_2 = qEu_0\tau \sin(\Gamma+\phi)\sin\Gamma/\Gamma$ considerably enhances the sensitivity. The bandwidth of the detector is determined by the term $\sin\Gamma/\Gamma$ and this is narrow and magnetically tunable. The system is therefore a spectrometer–detector.

In any sensitive detector the ultimate limit of detectivity set by noise fluctuations is important. In the cyclotron resonance detector this question is particularly interesting because the contributions from room temperature fluctuations are reduced by the narrow input bandwidth, and because the natural postdetector amplifier is a relatively noiseless electron multiplier. In the present paper we calculate the orders of the levels of noise originating from the various sources of random fluctuations. Four sources of noise are distinguished: (i) background room temperature radiation noise; (ii) detector noise ascribed to fluctuations of induced transitions, and photons emitted spontaneously by the cyclotron oscillators within the resonator; (iii) noise in the collected electron beam; (iv) postdetector amplifier noise.

Certain aspects of the noise (eg fluctuations due to spontaneous transitions) can be seen most clearly from a quantum mechanical viewpoint. Accordingly we give in the Appendix an outline of the origin of expression (1) in quantum theory. In short, the operation of the detector can be described in terms of induced transitions between Landau levels with the eigenstate energies of cyclotron motion $W_c = (n+\frac{1}{2})\hbar\omega_c$, where $h = 2\pi\hbar$ is Planck's constant and $n = 0, 1, 2, \ldots$. Electrons are initially pumped into the energy eigenstate $n = mu_0^2/2\hbar\omega_c$ by the configuration of magnetic fields. The interaction of the cyclotron resonance oscillators with a radiation field then distributes the electrons sinusoidally within the Landau ladder of states with amplitude

$$n = \frac{qEu_0\tau}{\hbar\omega_c}\frac{\sin\Gamma}{\Gamma}$$

and raises the mean level of energy by $\delta n = W_1/\hbar\omega_c$. Detection occurs when, as illustrated in figure 1, electrons from the highest populated levels overcome the applied bias potential barrier. This is essentially the picture developed in the Appendix, and used in § 3.

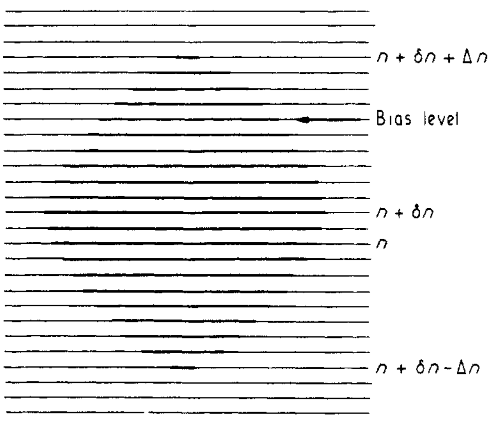

Figure 1. Illustration of the distribution of cyclotron resonance oscillators in the Landau ladder of eigenstates after interaction with a radiation field. The levels are separated by $\hbar\omega_c$, and the dark line on them represents the number of oscillators in the level. Initially all oscillators occupy the nth level, but the field perturbation shifts them upwards by δn and distributes them sinusoidally about $n+\delta n$ with amplitude Δn.

2. Radiation noise

A limitation to the minimum signal detectable by the cyclotron resonance mechanism will be set by random fluctuations in the stream of photons entering the resonator from the room temperature background. To estimate this we calculate the flux of photons crossing an area A (equal to the receiving aperture of the detector) per second within an enclosure in thermal equilibrium at temperature $T = 300$ K. The minimum detectable signal set by this source of noise we then take as equal to the statistical variation in this number of photons (the one second interval being the integration or observation time).

It is known from the statistical mechanics of Bose–Einstein particles that the mean volume density of photons in the frequency interval between ν and $\nu + \mathrm{d}\nu$ is

$$\bar{n} = \frac{8\pi\nu^2\,\mathrm{d}\nu}{c^3\{\exp(h\nu/kT)-1\}} \tag{2}$$

and that the mean square statistical fluctuation in this number density is

$$\overline{\Delta n^2} = \bar{n}\left(1+\frac{1}{\exp(h\nu/kT)-1}\right). \tag{3}$$

The application of equation (3) to the fluctuations in a stream of thermal radiation such as we are considering here has been discussed by Lewis (1947). The number N of photons crossing area A within solid angle $\mathrm{d}\Omega$ in a direction making angle θ with the normal to the area in time t is $\bar{n}\,cAt\cos\theta\,\mathrm{d}\Omega/4\pi$, and the mean square fluctuation in this number is $\overline{\Delta n^2}\,cAt\cos\theta\,\mathrm{d}\Omega/4\pi$. By (2) and (3) this is

$$\overline{\Delta N^2} = \frac{2\nu^2\exp(h\nu/kT)A\cos\theta\,\mathrm{d}\Omega\,\mathrm{d}\nu}{c^2\{\exp(h\nu/kT)-1\}^2}t.$$

For far infrared wavelengths approaching 1000 μm, $h\nu \ll kT$, and this becomes

$$\overline{\Delta N^2} = \frac{2k^2T^2A\cos\theta\,\mathrm{d}\Omega\,\mathrm{d}\nu}{c^2h^2}t.$$

Assume a detector aperture with $A\cos\theta = 1\ \mathrm{cm}^2$, and take $\mathrm{d}\Omega = 0{\cdot}2$ for the solid angle from which room temperature radiation is accepted. The root mean square fluctuation in the background photon count with only one polarization is

$$\frac{1}{2}(\overline{\Delta N^2})^{1/2} = \sqrt{0{\cdot}2}\,\frac{kT\,\mathrm{d}\nu^{1/2}}{ch}t^{1/2}. \tag{4}$$

Note here that the photon flux has two orthogonal polarizations of which only one can couple to the electric dipole moment of the cyclotron oscillator. Thus for comparison with a linearly polarized wanted signal, $\bar{N}$ (and so $\overline{\Delta N^2}$) must be halved. For $T = 300$ K and a detector input bandwidth $\mathrm{d}\nu = 10^7$ Hz as previously estimated (Robinson 1970a)

$$\tfrac{1}{2}(\overline{\Delta N^2})^{1/2} \simeq 3\times10^5\,t^{1/2}\ \text{photons}.$$

By our criterion of detectability we conclude that 3×10^5 linearly polarized signal photons can be detected in an observation time of 1 s. For a wavelength of 1000 μm this is a signal power of about 6×10^{-17} W. To calculate the energy imparted to the cyclotron oscillators by this signal we use (1). We have previously (Robinson 1970a) taken the interaction time τ as equal to 10^{-7} s, and have estimated that $E \simeq 3\times10^5\,P^{1/2}\ \mathrm{V\,m^{-1}}$

or $2{\cdot}3 \times 10^{-3}$ V m^{-1} for $P = 6 \times 10^{-17}$ W. Near the optimum phase angle the energies acquired by the cyclotron resonance oscillators will, by (1), be 7 to 8 quanta for 500 eV electrons and 1 or 2 quanta for 20 eV electrons.

In assuming $d\Omega = 0{\cdot}2$ sr we have implied that the resonator surrounding the electron beam is cooled well below room temperature. If the resonator is at room temperature the root mean square photon fluctuation, and hence the minimum detectable signals, will be increased by nearly an order of magnitude.

3. Detector noise

Detector noise may arise from fluctuations in the number of transitions between Landau levels induced by the input signal, and from spontaneous transitions.

That the former process is negligible can be seen immediately. If 500 eV electrons are injected into the resonator at the rate of 10^7 s^{-1} an input power of 6×10^{-17} W will result in a very large number ($\simeq 10^6$) acquiring 7 or 8 quanta of additional energy (those for which $\Gamma + \phi$ is in the neighbourhood of the optimum phase angle $\pi/2$ in equation (1)) to overcome the potential barrier. The ratio of this large number of collected electrons to its RMS fluctuation represents a large signal to noise ratio.

The probability of spontaneous emission of a photon by a cyclotron oscillator during its interaction time $\tau \simeq 10^{-7}$ s in the resonator can be found from well known expressions. The probability of spontaneous emission by an electron in the nth Landau level is (Louisell 1964)

$$w_{\text{spon}} = \frac{\omega_c^3}{3\pi c^2 \hbar}\left(\frac{\mu_0}{\epsilon_0}\right)^{1/2} \langle n-1|\mu|n\rangle \qquad (5)$$

where the matrix element is given by (7) in the Appendix, and μ_0 and ϵ_0 are the permeability and permittivity of free space.

If the initial kinetic energy is 500 eV, Landau levels with $n \simeq 4 \times 10^5$ will be populated. This quantum number may be used to calculate w_{spon} since, as can be seen in the Appendix, the perturbations δn and Δn produced by the interaction with radiation are relatively small. Again assuming a wavelength of 1000 μm we find

$$w_{\text{spon}} \simeq 3 \times 10^6 \text{ s}^{-1}.$$

Thus during the 10^{-7} s interaction time an electron emits a photon spontaneously with probability 0·3. This is to be compared with the 7 to 8 photons absorbed by induced transitions with 6×10^{-17} W of signal. This means that while induced transitions will cause favourably phased electrons to ascend 7 or 8 steps up the Landau ladder only 0·3 of these electrons will jump down a single level due to spontaneous emission of a photon. When 20 eV electrons are used $w_{\text{spon}} \simeq 6 \times 10^5$ s^{-1} and a cyclotron oscillator will emit a photon spontaneously with probability 0·06, which is much less than the 1 or 2 photons of emissions induced by 6×10^{-17} W.

4. Beam noise and postdetector amplifier noise

From our estimates in §§ 2 and 3 some 10^6 electrons should be detected under conditions where the input signal is approaching the limit set by background room temperature fluctuations. The RMS fluctuation of 10^3 electrons in this current (so called 'shot' noise) will be a negligible contribution to the overall noise. Fluctuations of this order will also prevail if a steady fraction of the current flows to the collector in the absence of radiation.

In addition to beam shot noise, flicker noise produced by fluctuations in the cathode emission is possible. Flicker noise can be minimized by chopping the signal at a high frequency f, for it falls off approximately as $f^{-1/2}$. The short response time of the cyclotron resonance detector makes it suitable for fast chopping.

During postdetector amplification additional noise will be introduced. However, the natural amplifier for a low current beam is an electron multiplier, and these are quoted as contributing very little noise. We can control the velocity of the electrons striking the first dynode by suitably biased grids and can thus adjust for maximum secondary emission. Fluctuations will arise in the number of secondary electrons emitted by the dynodes, but according to McLean and Putley (1965) this will increase the noise by only about 30–40%.

Additional noise may also come from fluctuations in the potentials of bias grids that arise from imperfect vacuum conditions. While this source of noise is practical rather than fundamental, it is clear from our foregoing figures that such fluctuations will become significant when they are at the millivolt level.

5. Concluding remarks

The noise equivalent power (NEP) of the cyclotron resonance detector appears to be close to the limit set by the fluctuations in background radiation calculated in § 2. By convention, NEP is defined for an output bandwidth of 1 Hz (which is an observation time of $t = 1/2\pi$ s) and we must increase the 6×10^{-17} W estimate for $t = 1$ s accordingly. Noise energy fluctuations (see equation (4)) vary as $t^{1/2}$ so for an observation time of $t \neq 1$ s the noise equivalent power is multiplied by a factor of $t^{-1/2}$. Thus for $t = 1/2\pi$ s the NEP becomes $1 \cdot 5 \times 10^{-16}$ W Hz$^{-1/2}$. The Jones (1953) factor of merit D^* (the reciprocal NEP for a detector aperture of 1 cm^2) is about 7×10^{15} cm Hz$^{1/2}$ W^{-1}.

Our estimates suggest an ultimate sensitivity for the cyclotron resonance detector some orders of magnitude better than both the indium antimonide free carrier far infrared detector and the Josephson junction. In its tuned mode the former has a NEP of about 5×10^{-11} W Hz$^{-1/2}$ (Putley 1963), and the latter has a limit in the region of 10^{-14} W Hz$^{-1/2}$ (*Physics Today* 1970). In the indium antimonide detector the major source of noise is the postdetector amplifier and this, as we suggested in § 4, is essentially eliminated when our electron beam is amplified in an electron multiplier. Background radiation is significant in both the indium antimonide and Josephson effect detectors, and it is here that the cyclotron resonance process under discussion offers a clear improvement through its narrow input bandwidth. It is narrower by a factor of 10^5 than the indium antimonide detector and it thereby reduces the background photon fluctuations by a factor of about 300. A similar improvement factor applies relative to the niobium–niobium Josephson junction described by Grimes *et al* (1966) and measured as having 5×10^{-13} W Hz$^{-1/2}$ NEP.

Recent work concerned with the storage of ions and electrons for precision hyperfine structure and g factor measurements (Wesley and Rich 1970 and private communication, Major and Dehmelt 1968) has shown that interaction times many orders of magnitude longer than the 10^{-7} s value assumed here can be attained. By using these techniques further reduction in input bandwidth is possible, and the ultimate sensitivity to small signals may be well below 10^{-16} W Hz$^{-1/2}$. An upper limit in electron storage time will be set by a compromise with the number of electrons stored in accordance with the need to keep the effects of space–charge forces small. Ultimately, the minimum cyclotron resonance linewidth will be set by such factors as the maximum attainable magnetic field homogeneity and the spread in the separations of populated Landau levels due to the relativistic energy dependence of electron mass.

Finally, a comment concerning the possibility of superheterodyne cyclotron resonance detection may be of interest. If a local oscillator wave is supplied to the resonator, then through the term in E^2 in equation (1) terms in W with sum and difference frequencies occur with amplitudes proportional to the product of the local oscillator and signal amplitudes. This can increase the magnitude of W_1 and its response to the signal, and we can physically interpret this as occurring because the local oscillator excites electrons to rise higher up the Landau ladder where (by (7), Appendix) they are more responsive to the input signal. However, the term W_2 in equation (1) already does exactly this through the term u_0, the initial velocity of the electrons. In this sense, we can regard the cyclotron resonance detector as having a superheterodyne principle inbuilt (the 'difference' or 'intermediate' frequency can be simulated by chopping the radiation, or by chopping the electron beam with an alternating voltage applied to a grid).

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Appendix. Quantum mechanical description of the cyclotron resonance interaction

It is easy to show (Landau and Lifshitz 1965), and it is well known, that the Hamiltonian operator for an electron in a homogeneous magnetic field results in a form of the Schrödinger equation that is formally identical with that of the harmonic oscillator. The kinetic energy of the cyclotron motion therefore has quantized values given by $(n+\frac{1}{2})\hbar\omega_c$, and forms the Landau ladder of states specified by $n = 0, 1,$ etc. Drift motion parallel to the magnetic field adds to this an energy continuum. The wavefunctions corresponding to the energy eigenstates are the well known 'number' eigenfunctions of Hermite polynomials.

If we regard the interaction between the electron and the electric fields as a switched-on perturbation of duration τ and assume nondegeneracy of the eigen energy states, we can apply the results of first order perturbation theory to the interaction. Electrons with kinetic energy $\frac{1}{2}mu_0^2$ are 'pumped' into the nth Landau level prior to the interaction and undergo induced transitions for single quantum absorption to the $(n+1)$th state with probability (Davydov 1965)

$$w_{n,n+1} = w_{n+1,n} = \frac{E^2}{\hbar^2}|\langle n|\mu|n+1\rangle|^2 g_\omega(\omega_c). \tag{6}$$

The matrix element of the electric dipole moment is (Davydov 1970)

$$\langle n|\mu|n+1\rangle = q\left(\frac{(n+1)\hbar}{2m\omega_c}\right)^{1/2} \tag{7}$$

and the lineshape function $g_\omega(\omega_c)$ is given by

$$g_\omega(\omega_c) = \tau^2 \frac{\sin^2\Gamma}{\Gamma^2}. \tag{8}$$

Replacement of n by $n-1$ in (6) and (7) gives the probability of induced emission. For an nth state population of N electrons the net cyclotron energy absorbed by induced transitions in time τ is

$$N\hbar\omega_c(w_{n+1,n} - w_{n-1,n}) = \frac{Nq^2E^2\tau^2}{2m}\frac{\sin^2\Gamma}{\Gamma^2}. \tag{9}$$

Comparing (9) and (1) we observe that first order perturbation theory describing single quantum transitions yields the phase averaged energy absorption W_1.

Electron motion with phase ϕ with respect to the driving force can be interpreted in terms of a wave packet formed from a superposition of wavefunctions of number eigenstates. A gaussian wave packet satisfying the Heisenberg minimum uncertainty condition and whose centre moves in cyclotron motion precisely as the classical particle (Henley and Thirring 1962) can be constructed. The state constructed by the superposition is called the coherent state. The forced motion of the wave packet can be analysed in terms of coherent states using an operator algebra that has been described by a number of writers (Louisell 1964, Glauber 1963, Fuller *et al* 1963, Carruthers and Nieto 1965).

We seek answers to the following questions: how are N cyclotron harmonic oscillators, initially in the nth energy eigenstate, redistributed in the Landau ladder by the interaction described? How does the quantum description of the energy redistribution go over to the classical limit? To answer these questions we apply the analysis of Carruthers and Nieto (1965) for the coherent states of a forced harmonic oscillator.

Coherent states $|\alpha\rangle$ are eigenfunctions of the annihilation operator a in the sense that

$$a|\alpha\rangle = \alpha|\alpha\rangle \tag{10}$$

where

$$\alpha = |\alpha|\, e^{i\phi} \tag{11}$$

is a complex number whose argument is the phase angle of the oscillator, and the square of whose modulus gives the mean excitation of the oscillator. The Hamiltonian operator can be written

$$H = \hbar\omega_c(a^+a + \tfrac{1}{2}). \tag{12}$$

For a harmonic oscillator in an initial state specified by a (and a^+), and raised by a time dependent driving force with Fourier transform F to a final state specified by

$$b = a + i\alpha_0 \tag{13}$$

where

$$\alpha_0 = \frac{F}{(2m\hbar\omega_c)^{1/2}} \tag{14}$$

Carruthers and Nieto derive an operator S (the S matrix) in terms of which the matrix element $\langle k|S|n\rangle$ for transitions from an initial number state n to any final number state

k can be found. For the present case of a time dependent force acting on the harmonic oscillator (Carruthers and Nieto 1965)

$$S = \exp\left(\frac{\mathrm{i}(a^+F + aF^*)}{(2m\hbar\omega_c)^{1/2}}\right). \tag{15}$$

It can be shown (Carruthers and Nieto 1965) that this leads to the transition probability

$$w_{k,n} = |S_{kn}|^2 = \frac{n!}{k!}\exp(-|\alpha_0|^2)|\alpha_0|^{2(k-n)}(L_k^{k-n}(|\alpha_0|^2))^2 \tag{16}$$

for $k \geqslant n$, or by the same formula but with k and n interchanged if $k < n$. The term $L_k^{k-n}(|\alpha_0|^2)$ is the associated Laguerre polynomial.

Fuller *et al* (1963) have summed (16) over upward transitions $k > n$, and downward transitions $k < n$, and have shown that the mean energy transfer to an oscillator is $\hbar\omega_c|\alpha_0|^2$. This is precisely the phase averaged term W_1 in the classical expression (1). Carruthers and Nieto have obtained the same result for the average energy absorbed, but their formulation also yields the phase dependent term. In the Heisenberg picture of quantum mechanics we can write the energy shift produced by the interaction as

$$W = \langle\Psi_{\text{in}}|H_{\text{out}} - H_{\text{in}}|\Psi_{\text{in}}\rangle. \tag{17}$$

Here the subscript 'in' refers to the initial state of the oscillator and the subscript 'out' refers to the state of the oscillator after the interaction. The Hamiltonian operator of the 'in' state oscillator is

$$H_{\text{in}} = \hbar\omega_c(a^+a + \tfrac{1}{2}) \tag{12}$$

and that of the 'out' state is

$$H_{\text{out}} = \hbar\omega_c(b^+b + \tfrac{1}{2}) \tag{18}$$

where the annihilation operators are related as in (13). Substitution gives

$$W = \hbar\omega_c\{|\alpha_0|^2 + \mathrm{i}(\langle\Psi_{\text{in}}|a^+|\Psi_{\text{in}}\rangle\alpha_0 - \text{complex conjugate})\}. \tag{19}$$

If Ψ_{in} is a coherent state with phase parameter $\beta = |\beta|\,\mathrm{e}^{\mathrm{i}\phi}$, (19) reduces to

$$W = \hbar\omega_c(|\alpha_0|^2 + 2\,\mathrm{Im}(\beta\alpha_0^*)). \tag{20}$$

In going over to the classical cyclotron oscillator we must have

$$|\beta| = \left(\frac{m\omega_c}{2\hbar}\right)^{1/2} r_{\text{c out}} \tag{21}$$

where $r_{\text{c out}}$ is the cyclotron orbit radius at the termination of the interaction. For a constant driving field of angular frequency ω and duration τ, $F = qE\tau \sin\Gamma/\Gamma$ in (14). When α_0^* and β are substituted from (14) and (21), equation (20) reduces exactly to the classical expression (1).

The detector operates as follows: electrons are initially pumped into the nth Landau level where $n = mu_0^2/2\hbar\omega_c$. While raising the mean level of the system by an amount $\delta n = W_1/\hbar\omega_c$, the interaction with the radiation field distributes the electrons about the level $n+\delta n$ in a sinusoidal distribution up to the peak level $n+\delta n+\Delta n$, and down to the level $n+\delta n-\Delta n$, where $\Delta n = (qEu_0\tau/\hbar\omega_c)(\sin\Gamma/\Gamma)$. Only those electrons from the highest populated levels of the Landau ladder can overcome the applied bias potential barrier and be detected. The situation is illustrated diagramatically in figure 1. This 'clipping' of oscillators from the vicinity of the peak of the distribution of populated number states provides the nonlinearity of the detection process. The separation of the Landau levels $\hbar\omega_c$ is magnetic field tunable and the width of the levels is given by the $\sin^2\Gamma/\Gamma^2$ function.

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